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Original Article

Enhancing Fermatean Fuzzy Transportation Problems: An Innovative Score Function-Based Optimization Strategy

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Abstract: *This research paper introduces a novel approach for optimizing Fermatean fuzzy transportation problems (FFTP) by integrating a unique score function. Fermatean fuzzy numbers (FFN), known for their ability to represent uncertainty, pose challenges in transportation optimization. The proposed score function addresses these challenges by providing a comprehensive evaluation metric. We developed a traditional transportation problem (TP) model with a fermatean fuzzy environment (FFE) and, utilizing a new score function, transformed it into the deterministic form. Then, using the expected value technique, we created a new multiobjective, multi-level solid transportation model (MOMLST) using FFE and translated it into crisp form. Again, develop a multiobjective, multi-level solid transportation problem with fermatean fuzzy parameters (MOMLSTPWFF). It cannot be directly optimized because the fermatean fuzzy parameters (FFP) exist in levels of objective functions and are subject to constraints. However, we will follow the new score function in FFE and convert the mathematical model into crisp form. The numerical example is also provided to justify the convenience of the MOMLSTPWFF mathematical model and find a TP strategy that is best for our proposed mathematical model.*

Keywords: *Fermatean Fuzzy Transportation Problems, Optimization, New Score Function, Uncertainty Modeling, Multiobjective Optimization.*

I. INTRODUCTION

The fundamental TP was initially proposed by (Hitchcock, 1941) 1941 as a means to carry goods from numerous sources to numerous locations. In 1949, (Koopmans 1949) published his influential paper "Optimum Utilization of the Transport System," wherein he presented several methods for solving TP. Subsequently, in 1963, (Dantzig 1963) developed the simplex method in linear programming to address TP. However, this method often proves to be time-consuming due to the involvement of many variables and constraints, making it challenging to obtain the optimal solution efficiently. To overcome these challenges, researchers have since devised alternative methods, such as the (NWCM), (LCM), and (VAM). These methods aim to optimize the TP and suggest optimal solutions for these problems as a starting point for subsequent optimization. By exploring these alternative approaches, researchers have sought to improve the efficiency and effectiveness of solving TP, ultimately facilitating the identification of optimal solutions. The basic idea of fuzzy sets was developed by (Zadeh, 1965) in 1965 and further extended by (Goguen, 1967) in 1967.

The TP was initially developed by Hitchcock (1941) and deals with transporting products from suppliers to final locations. An extension of TP is the STP, which considers three kinds of constraints instead of two, such as modes of transportation or types of goods. In many real-world situations, multiple objectives must be considered and optimized simultaneously, leading to a multiobjective problem (Kundu et al., 2014). Transportation frameworks are used in supply chain management, and logistics play a pivotal role by enhancing efficiency and service quality. The STP gains relevance in scenarios with diverse modes and flexible routes, optimizing costs and delivery times. Prior studies have examined this problem, drawing comparisons with the classical variant and employing fuzzy linear programming to tackle multiobjective challenges (Liu, 2006). (Tao and Xu, 2012) introduced the notion of a rough multiple-objective programming model tailored to address the intricacies of an STP. The problem variant encompasses crucial constraints associated with source availability, destination requirements, and conveyance capacity. They highlight the increasing scholarly attention toward multiple objectives, STP.

(Kundu et al., 2013) tackle a multiobjective STP involving fragile goods, optimizing cost and time. The analysis incorporates conveyance type and quantity as factors impacting fragility. The stochastic nature of destination demand, influenced by breakability, is effectively addressed through the chance-constraint method. (Pramanik et al., 2013) represented the classical TP as a subset of linear programming, encompassing diverse conveyances for homogeneous product delivery. It highlights the challenge of integrating multiple conflicting objectives in real-world scenarios, leading to STP with restricted route capacities. They propose mitigating uncertainty by adopting flexible, imprecise fuzzy numbers representing key parameters like penalties, demands, availabilities, and capacities. (Revathi et al., 2021) focused on maximizing profit by considering numerous objectives because the TP with a single aim is insufficient in the current market. The research notes that the cost parameters of distinct purposes are unrelated, making them incompatible and comparable. Fuzzy sets and fuzzy numbers are mentioned to deal with uncertainty in the TP. To show the effectiveness of fuzzy linear programming solutions, (Zimmermann, 1978) addresses the linear vector maximum problem using fuzzy linear programming techniques. They discuss the effects of using different techniques to combine distinct objective functions to find the best solution. They also provide valuable insights into the effectiveness of fuzzy linear programming in tackling multiobjective problems and offer guidance on selecting appropriate approaches for achieving optimal compromise solutions.

This work's remaining sections are arranged as follows: The literature study is presented in Section 2, and various arithmetic operations, theorems, and basic definitions of the FFPA are shown in Section 3. In section 4, we built a general classical transportation model with FFP, the MOMLSTP model, the MOMLUSTP model, and MOMLSTPWFF. We converted these models into a crisp form using different mathematical programming approaches. Section 5 depicts the solution procedure of MOMLSTPWFF. In Section 6, we provide numerical example results to testify to the efficiency of the suggested method. At last, in Section 7, the findings are summarized.

II. LITERATURE REVIEW

(Chanas & Kuchta, 1996) provided a method created expressly to find the TP ideal resolution when considering uncertain variables. Their approach draws from the generic fuzzy coefficient linear programming issue, offering a practical framework to handle the uncertainties inherent in transportation optimization. It enhances decision-making processes by incorporating fuzziness in cost coefficients and providing a robust algorithm to determine optimal solutions. (Roy et al.,2019) focused on a specific type of TP known as the multiobjective multi-item inflexible STP model under uncertain parameters. To address this problem, they employed fuzzy goal programming techniques and fuzzy programming to find solutions that effectively handle uncertainty. (Sergi & Sari, 2021) introduced FFSs as a relatively new addition to fuzzy set theory. These sets offer the flexibility to assign fuzzy parameters from a broader domain, enabling the management of higher degrees of uncertainty. This advancement in fuzzy set theory provides a more comprehensive framework for handling uncertainty in various applications. (Sahoo,2021) proposed a novel score function utilizing FFSs was introduced to address TP influenced by uncertain economic and environmental factors, where precise measurements of supply, demand, and transportation costs are challenging. The proposed algorithm leverages arithmetic operations of FFN to achieve the best solutions for TP with uncertain parameters. (Sharma et al., 2022) emphasized addressing uncertainty in competitive market conditions regarding transportation costs, supply, and demand factors. To address impreciseness in TP, they propose the FFSs as an extension of the Pythagorean fuzzy set. This novel approach aims to effectively handle the volatility of market conditions and enhance decision-making processes in transportation logistics.

(Nagar et al., 2022) presented a novel approach for optimizing a specific class of Pythagorean fuzzy TP by introducing a new score function. This innovative method incorporates hesitation information, enabling more accurate analysis of Pythagorean fuzzy numbers. The proposed score function enhances the precision and reliability of decision-making processes in transportation optimization. (Revathi and Mohanaselvi, 2021) introduce a comprehensive model for addressing a four-dimensional multiobjective multi-item fractional TP in uncertainty. A proposed model incorporates uncertain variables in objectives and constraints, making it well-suited for real-life scenarios. By employing models of chance and anticipated value constraints grounded in uncertainty theory, they successfully derive an equivalent deterministic model for effectively tackling the uncertainties in the problem. (Jalil et al., 2018) proposed a comprehensive multiobjective mathematical model with fuzzy parameters. They introduce a fuzzy programming technique specifically tailored for solving multiobjective mathematical problems, offering an effective solution for the presented model and enhancing decision-making processes in solid transportation logistics. (Singh et al., 2019) introduced a solution methodology utilizing chance-constraint programming to address the STP with uncertainty characterized by the gamma distribution. Their approach extends the fuzzy programming technique to effectively solve multiobjective stochastic problems, providing a robust framework for handling STP uncertainty. This methodology enhances decision-making processes in transportation logistics by considering probabilistic constraints and offering a comprehensive approach to address multiobjective uncertainties. (Gupta & Kumar, 2012) identify limitations in the current approach for solving linear multiobjective TP with unknown variables and propose a new method to address these shortcomings. (Dalman, 2018) introduced the deterministic mathematical model into a single objective mathematical model through the convex combination method and the minimizing distance function method, providing practical tools for addressing the TP and facilitating decision-making in solid transportation logistics. They convert the uncertain mathematical model into an equivalent deterministic model by leveraging the advantages of uncertainty theory.

(Senapati & Yager, 2020) introduced FFSs as an extension of intuitionistic fuzzy sets, providing a comprehensive comparison with Pythagorean and intuitionistic fuzzy sets. They delve into the fundamental properties of FFSs, including the complement operator and the entire set of operations. (Silambarasan, 2020) examined the algebraic properties of these operators, providing valuable insights into the mathematical foundation of FFSs. They expand the theoretical framework of FFSs and enrich the understanding of their operational characteristics. (Akram et al., 2022) presented the concept of interval-valued FFSs as a robust approach for handling uncertain and incomplete data.

Additionally, they propose a novel method for directly addressing interval-valued Fuzzy Fermatean fractional TP, avoiding the need to convert the original problem into a crisp equivalent, thereby streamlining the solution process. It enhances the resilience and efficiency of addressing uncertainties in transportation logistics. (Niksirat, 2022) conducted a comprehensive investigation into the fully fuzzy multiobjective transportation (FFMOT) problem. They presented a novel approach utilizing the closest approximation for an interval to solve the FFMOT problem effectively, enabling the determination of Pareto optimal solutions even under conditions of uncertainty. It contributes to advancing the field of multiobjective transportation optimization by addressing the challenges posed by fuzziness and uncertainty, thereby facilitating informed decision-making in TP. The multiobjective transportation p-facility location problem is addressed by (Das & Roy 2019) using a novel technique that integrates the TP with the facility location problem within a multiobjective framework. Combining using a neutrosophic compromises programming and an alternate locate-allocate strategy to obtain nondominated solutions creates a hybrid strategy that offers a thorough solution methodology for this challenging problem area.

(Li & Lai, 2000; Singh & Yadav, 2016) presented a fuzzy optimization technique for multiobjective TP, considering multiple objectives with both individual marginal evaluation and global evaluation for all objectives. They provide a comprehensive methodology for decision-making in multiobjective transportation optimization, offering a balanced and effective approach to address conflicting objectives. (Gul et al., 2021) provided a unique technique for evaluating occupational risks in the industry utilizing FFSs and the TOPSIS method. FFSs, an advanced version of fuzzy set theory, encompass intuitionistic and Pythagorean fuzzy sets, enabling efficient handling of uncertain information and improved representation of uncertainty. By converting them into balanced counterparts, (Shivani et al., 2022) presented a novel strategy for handling totally rough, imbalanced, multiobjective, fixed-charge TP. They use fuzzy programming, goal programming, and the weighted-sum method to improve transportation logistics decision-making. They provide Pareto-optimal solutions for the transformed balanced, completely rough multiobjective fixed-charge TP. Their recent study (Akram et al., 2023) introduced a novel approach called the Fermatean fuzzy DEA method aimed at addressing the Fermatean fuzzy multiobjective TP. By transforming the FFMOTP into a single objective Fermatean fuzzy TP, they demonstrate that standard algorithms can effectively solve it. (Sharma et al., 2023) presented FFPA as a novel methodology to address multiobjective TP within a Fermatean fuzzy environment (FFE). They introduced a novel score function based on the Fermatean fuzzy technique, which effectively converts fuzzy data into crisp data, enabling more precise analysis and decision-making. (Akram et al., 2023) proposed FFSs as a superior and versatile model for handling uncertainty compared to Pythagorean fuzzy sets. They examine the multiobjective TP inside a framework of ranking triangular FFN, a new consensus technique. Additionally, they provide a solution procedure that effectively solves a real-world problem, showcasing the practical applicability and efficacy of the proposed method.

Table 2: Some Important Research Works on Different Variants of the TP and STP

S.No.	References	Years	Environments
1	(Sakawa, 1984)	1984	Multiobjective Nonlinear Programming Problems to Interactive Fuzzy Goal Programming
2	(Bit et al., 1992)	1992	Fuzzy programming technique for decision-making based on multiple criteria in transportation problems
3	(Abd El-Wahed & Lee, 2006)	2006	Multiobjective transportation problems under Interactive fuzzy goal programming
4	(Moanta, 2007)	2007	Using a simplex algorithm to solve the linear fractional transportation problem
5	(Korukoğlu & Ballı, 2011)	2011	Using VAM in TP
6	(Gupta & Kumar, 2012)	2012	Fuzzy programming approach used in linear multiobjective transportation problems
7	(Kundu et al., 2013)	2013	Using fuzzy programming multiobjective multi-item STP
8	(Bharati & Singh, 2018)	2018	Transport issue: interval-valued intuitionistic fuzzy sets
9	(Mahmoodirad et al., 2019)	2019	Fully Intuitionistic fuzzy programming approach to the transportation problem
10	(Senapati & Yager, 2019)	2019	Fermatean fuzzy weighted geometric operator in MCDM
11	(Das et al.,2020)	2020	Using the type-2 fuzzy logic approach in MOGSTLP
12	(Ghosh et al., 2021)	2021	Fully intuitionistic fuzzy programming approach in multiobjective, fully fixed-charge solid transportation problem
13	(Midya et al., 2021)	2021	Intuitionistic fuzzy parameters approach used to MSMOFSTP in the green supply chain.

14	(Sahoo, 2021)	2021	Fermatean fuzzy parameters in TP
15	(Ghosh et al., 2022)	2022	MOSTP in the Pythagorean fuzzy environment
16	(Giri & Roy, 2022)	2022	Neuromorphic programming techniques used in MOGFDFTP
17	(Bagheri et al., 2022)	2022	A data envelopment analysis approach is proposed to solve the (FMOTP)
18	(Bera & Mondal, 2022)	2022	Gaussian fuzzy programming approach used in MOTP
19	(Ghosh et al., 2022)	2022	MOSTP to Preservation Technology
20	(Sharma et al., 2023)	2023	Fermatean fuzzy new score function used in solid transportation problem

III. PRELIMINARIES AND DEFINITIONS

Basic definitions of the farmatean fuzzy programming, which are used in our proposed work, which is given below:

Definition 3.1 "(Senapati & Yager, 2020) farmatean fuzzy sets: A farmatean fuzzy sets (FFSs) can be represented as $\tilde{\mathcal{F}} = \{\langle \omega, \alpha_{\tilde{\mathcal{F}}}(\omega), \beta_{\tilde{\mathcal{F}}}(\omega) : \omega \in X \rangle\}$

Where $\alpha_{\tilde{\mathcal{F}}}(\omega): X \to [0,1]$ is the degree of satisfaction and $\beta_{\tilde{\mathcal{F}}}(\omega): X \to [0,1]$ is the degree of dissatisfaction, including the condition

$$0 \le \alpha_{\tilde{\mathcal{F}}}(\omega)^3 + \beta_{\tilde{\mathcal{F}}}(\omega)^3 \le 1 \ \forall \ \omega \in X .$$

For any farmatean fuzzy sets (FFSs) $\tilde{\mathcal{F}}$ and $\omega \in X$,

$\sigma_{\tilde{\mathcal{F}}}(\omega) = \sqrt[3]{1 - (\alpha_{\tilde{\mathcal{F}}}(\omega))^3 - (\beta_{\tilde{\mathcal{F}}}(\omega))^3}$ is identified as the degree of indeterminacy of x to $\tilde{\mathcal{F}}$.The set $\tilde{\mathcal{F}} = \{\langle \omega, \alpha_{\tilde{\mathcal{F}}}(\omega), \beta_{\tilde{\mathcal{F}}}(\omega) : \omega \in X \rangle\}$ is denoted as $\tilde{\mathcal{F}} = \langle \alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \rangle$."

Definition 3.2 "Let $\tilde{\mathcal{F}} = \langle \alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \rangle$, $\tilde{\mathcal{F}}_1 = \langle \alpha_{\tilde{\mathcal{F}}_1}, \beta_{\tilde{\mathcal{F}}_1} \rangle$ and $\tilde{\mathcal{F}}_2 = \langle \alpha_{\tilde{\mathcal{F}}_2}, \beta_{\tilde{\mathcal{F}}_2} \rangle$ be three farmatean fuzzy sets (FFSs) on the universal set X, and $\zeta > 0$ be any scalar, then arithmetic operations of farmatean fuzzy sets is as follows with a numerical example.

2.21 $\tilde{\mathcal{F}}_1 \oplus \tilde{\mathcal{F}}_2 = \left(\sqrt[3]{\alpha_{\tilde{\mathcal{F}}_1}{}^3 + \alpha_{\tilde{\mathcal{F}}_2}{}^3 - \alpha_{\tilde{\mathcal{F}}_1}{}^3 \alpha_{\tilde{\mathcal{F}}_2}{}^3},\ \beta_{\tilde{\mathcal{F}}_1} \beta_{\tilde{\mathcal{F}}_2} \right)$.

Let $\tilde{\mathcal{F}} = \langle 0.4, 0.7 \rangle$, $\tilde{\mathcal{F}}_1 = \langle 0.8, 0.6 \rangle$ and $\tilde{\mathcal{F}}_2 = \langle 0.2, 0.9 \rangle$ be three farmatean fuzzy sets (FFSs), and $\zeta = 2$ be any scalar. Then

$\tilde{\mathcal{F}}_1 \oplus \tilde{\mathcal{F}}_2 = \langle 0.8, 0.6 \rangle \oplus \langle 0.2, 0.9 \rangle = (0.8020, 0.54)$

2.22 $\tilde{\mathcal{F}}_1 \otimes \tilde{\mathcal{F}}_2 = \left(\alpha_{\tilde{\mathcal{F}}_1} \alpha_{\tilde{\mathcal{F}}_2}, \sqrt[3]{\beta_{\tilde{\mathcal{F}}_1}{}^3 + \beta_{\tilde{\mathcal{F}}_2}{}^3 - \beta_{\tilde{\mathcal{F}}_1}{}^3 \beta_{\tilde{\mathcal{F}}_2}{}^3} \right)$.

$\tilde{\mathcal{F}}_1 \otimes \tilde{\mathcal{F}}_2 = \langle 0.8, 0.6 \rangle \oplus \langle 0.2, 0.9 \rangle = (0.16, 0.923)$

2.23 $\zeta \odot \tilde{\mathcal{F}} = \left(\sqrt[3]{1 - (1 - \alpha_{\tilde{\mathcal{F}}}{}^3)^\zeta}, \beta_{\tilde{\mathcal{F}}}{}^\zeta \right)$.

$\zeta \odot \tilde{\mathcal{F}} = 2 \odot \langle 0.4, 0.7 \rangle = (0.498, 0.49)$

2.24 $\tilde{\mathcal{F}}^\zeta = \left(\alpha_{\tilde{\mathcal{F}}}{}^\zeta, \sqrt[3]{1 - \left(1 - \beta_{\tilde{\mathcal{F}}}{}^3\right)^\zeta} \right)$.

$\tilde{\mathcal{F}}^\zeta = \langle 0.4, 0.7 \rangle^2 = (0.064, 0.828)$."

Definition 3.3 "Let $\tilde{\mathcal{F}} = \langle \alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \rangle$, $\tilde{\mathcal{F}}_1 = \langle \alpha_{\tilde{\mathcal{F}}_1}, \beta_{\tilde{\mathcal{F}}_1} \rangle$ and $\tilde{\mathcal{F}}_2 = \langle \alpha_{\tilde{\mathcal{F}}_2}, \beta_{\tilde{\mathcal{F}}_2} \rangle$ be three farmatean fuzzy sets (FFSs) on the universal set X and $\zeta > 0$ be any scalar, then the arithmetic operations of farmatean fuzzy sets are defined as follows.

2.31 $\tilde{\mathcal{F}}_1 \cup \tilde{\mathcal{F}}_2 = \left(\max\{\alpha_{\tilde{\mathcal{F}}_1}, \alpha_{\tilde{\mathcal{F}}_2}\}, \min\{\beta_{\tilde{\mathcal{F}}_1}, \beta_{\tilde{\mathcal{F}}_2}\} \right)$.

$\tilde{\mathcal{F}}_1 \cup \tilde{\mathcal{F}}_2 = (\max\{\langle 0.8, 0.6 \rangle\}, \min\{\langle 0.2, 0.9 \rangle\}) = (0.8, 0.9)$

2.32 $\tilde{\mathcal{F}}_1 \cap \tilde{\mathcal{F}}_2 = \left(\min\{\alpha_{\tilde{\mathcal{F}}_1}, \alpha_{\tilde{\mathcal{F}}_2}\}, \max\{\beta_{\tilde{\mathcal{F}}_1}, \beta_{\tilde{\mathcal{F}}_2}\} \right)$.

$\tilde{\mathcal{F}}_1 \cap \tilde{\mathcal{F}}_2 = (\min\{\langle 0.8, 0.6 \rangle\}, \max\{\langle 0.2, 0.9 \rangle\}) = (0.2, 0.6)$

2.33 $\tilde{\mathcal{F}}^c = (\beta_{\tilde{\mathcal{F}}}, \alpha_{\tilde{\mathcal{F}}})$.

$\tilde{\mathcal{F}}^c = \langle 0.4, 0.7 \rangle^c = (0.7, 0.4)$."

Accuracy Function of FFSs:

Suppose $\tilde{\mathcal{F}} = \langle \alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \rangle$ be an FFSs, then the accuracy function of FFSs is represented as follows,

$$A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}) = \left(\alpha_{\tilde{\mathcal{F}}}{}^3 + \beta_{\tilde{\mathcal{F}}}{}^3 \right).$$

Theorem 1

Let $\tilde{\mathcal{F}}$ be a FFSs $\tilde{\mathcal{F}} = \langle \alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \rangle$ then the score function $\tilde{\mathcal{F}}$ represented simply proceeds;

$$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}) = \frac{1}{2}\left(1 + \alpha_{\tilde{\mathcal{F}}}{}^3 - \beta_{\tilde{\mathcal{F}}}{}^3\right).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}})).$$

Property 1. Consider a FFSs $\tilde{\mathcal{F}} = \langle \alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \rangle$, then $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}) \in [0,1]$.

Proof: According to the ortho-pair definition, $\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \in [0,1]$. Then, $\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}) \in [0,1]$, and also

$$\alpha_{\tilde{\mathcal{F}}}{}^3 \ge o, \beta_{\tilde{\mathcal{F}}}{}^3 \ge 0, \alpha_{\tilde{\mathcal{F}}}{}^3 \le 1, \text{and } \beta_{\tilde{\mathcal{F}}}{}^3 \le 1$$

$$\Rightarrow 1-\beta_{\tilde{\mathcal{F}}}{}^3 \geq 0$$
$$\Rightarrow 1+\alpha_{\tilde{\mathcal{F}}}{}^3-\beta_{\tilde{\mathcal{F}}}{}^3 \geq 0$$
$$\therefore \frac{1}{2}\left(1+\alpha_{\tilde{\mathcal{F}}}{}^3-\beta_{\tilde{\mathcal{F}}}{}^3\right).(\min(\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}})) \geq 0$$
Again $\alpha_{\tilde{\mathcal{F}}}{}^3-\beta_{\tilde{\mathcal{F}}}{}^3 \leq 1$, add one both sides
$$\Rightarrow 1+\alpha_{\tilde{\mathcal{F}}}{}^3-\beta_{\tilde{\mathcal{F}}}{}^3 \leq 2 \qquad (\because \alpha_{\tilde{\mathcal{F}}}{}^3 \geq 0)$$
$$\Rightarrow \frac{1}{2}\left(1+\alpha_{\tilde{\mathcal{F}}}{}^3-\beta_{\tilde{\mathcal{F}}}{}^3\right).(\min(\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}})) \leq 1 \qquad (\because \min(\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}) \leq 1)$$
Hence, $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}) \in [0,1]$.

New Fermatean Fuzzy Score Function (NFFSF):

Theorem 2.

Let $\tilde{\mathcal{F}}$ be a FFSs $\tilde{\mathcal{F}}=\langle\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}\rangle$ then the (NFFSF) $\tilde{\mathcal{F}}_{1D}$ represented simply proceeds;
$$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{1D})=\frac{1}{2}(1+\alpha_{\tilde{\mathcal{F}}}-\beta_{\tilde{\mathcal{F}}}).(\min(\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}))^2$$
Property 1. Consider a FFSs $\tilde{\mathcal{F}}=\langle\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}\rangle$, then $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{1D}) \in [0,1]$.

Proof: According to the ortho-pair definition, $\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}} \in [0,1]$. Then, $\min(\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}) \in [0,1]$, and also
$$\alpha_{\tilde{\mathcal{F}}} \geq o,\ \beta_{\tilde{\mathcal{F}}} \geq 0,\ \alpha_{\tilde{\mathcal{F}}} \leq 1, \text{ and } \beta_{\tilde{\mathcal{F}}} \leq 1$$
$$\Rightarrow 1-\beta_{\tilde{\mathcal{F}}} \geq 0$$
$$\Rightarrow 1+\alpha_{\tilde{\mathcal{F}}}-\beta_{\tilde{\mathcal{F}}} \geq 0$$
$$\therefore \frac{1}{2}(1+\alpha_{\tilde{\mathcal{F}}}-\beta_{\tilde{\mathcal{F}}}).(\min(\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}))^2 \geq 0$$
Again, $\alpha_{\tilde{\mathcal{F}}} \leq 1$, and $\beta_{\tilde{\mathcal{F}}} \leq 1$, $\alpha_{\tilde{\mathcal{F}}}-\beta_{\tilde{\mathcal{F}}} \leq 1$,
And add one on both side
$$\Rightarrow 1+\alpha_{\tilde{\mathcal{F}}}-\beta_{\tilde{\mathcal{F}}} \leq 2 \ \Rightarrow (\min(\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}) \leq 1) \Rightarrow (\min(\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}))^2 \leq 1$$
$$\Rightarrow \frac{1}{2}(1+\alpha_{\tilde{\mathcal{F}}}-\beta_{\tilde{\mathcal{F}}}).(\min(\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}))^2 \leq 1 \qquad (\because (\min(\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}))^2 \leq 1)$$
Hence, $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{1D}) \in [0,1]$.

Theorem 2. Let $\tilde{\mathcal{F}}$ be a FFSs $\tilde{\mathcal{F}}=\langle\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}\rangle$ then the Type 1 score function $\tilde{\mathcal{F}}_1$ represented simply proceeds;

Type-1 fermatean fuzzy score function $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{11})=\frac{1}{2}\left(1+\alpha_{\tilde{\mathcal{F}}}{}^2-\beta_{\tilde{\mathcal{F}}}{}^2\right)$.

According to the ortho-pair definition, $\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}} \in [0,1]$, and
$$\alpha_{\tilde{\mathcal{F}}}{}^2 \geq o,\ \beta_{\tilde{\mathcal{F}}}{}^2 \geq 0,\ \alpha_{\tilde{\mathcal{F}}}{}^2 \leq 1, \text{ and } \beta_{\tilde{\mathcal{F}}}{}^2 \leq 1$$
$$\Rightarrow 1-\beta_{\tilde{\mathcal{F}}}{}^2 \geq 0$$
$$\Rightarrow 1+\alpha_{\tilde{\mathcal{F}}}{}^2-\beta_{\tilde{\mathcal{F}}}{}^2 \geq 0$$
$$\therefore \frac{1}{2}\left(1+\alpha_{\tilde{\mathcal{F}}}{}^2-\beta_{\tilde{\mathcal{F}}}{}^2\right) \geq 0$$
Now, again $\alpha_{\tilde{\mathcal{F}}}{}^2-\beta_{\tilde{\mathcal{F}}}{}^2 \leq 1$, add one both sides
$$\Rightarrow 1+\alpha_{\tilde{\mathcal{F}}}{}^2-\beta_{\tilde{\mathcal{F}}}{}^2 \geq 2 \qquad (\because \alpha_{\tilde{\mathcal{F}}}{}^2 \geq 0)$$
$$\Rightarrow \frac{1}{2}\left(1+\alpha_{\tilde{\mathcal{F}}}{}^2-\beta_{\tilde{\mathcal{F}}}{}^2\right) \geq 1 \qquad (\because \langle\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}\rangle \leq 1)$$
Hence, $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{11}) \in [0,1]$. Similarly,

Type-2 fermatean fuzzy score function $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{12})=\frac{1}{3}\left(1+2\alpha_{\tilde{\mathcal{F}}}{}^3-\beta_{\tilde{\mathcal{F}}}{}^3\right)$.

Type -3 fermatean fuzzy score function $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{13})=\frac{1}{2}\left(1+\alpha_{\tilde{\mathcal{F}}}{}^2-\beta_{\tilde{\mathcal{F}}}{}^2\right).|\alpha_{\tilde{\mathcal{F}}}-\beta_{\tilde{\mathcal{F}}}|$

"Let $\tilde{\mathcal{F}}_1=\langle\alpha_{\tilde{\mathcal{F}}_1},\beta_{\tilde{\mathcal{F}}_1}\rangle$ and $\tilde{\mathcal{F}}_2=\langle\alpha_{\tilde{\mathcal{F}}_2},\beta_{\tilde{\mathcal{F}}_2}\rangle$ be two farmatean fuzzy sets (FFSs), and then we have the following operation,
$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) \geq S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_2)$ with $A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_1) > A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_2)$ iff $\tilde{\mathcal{F}}_1 > \tilde{\mathcal{F}}_2$.
$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) \leq S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_2)$ with $A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_1) < A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_2)$ iff $\tilde{\mathcal{F}}_1 < \tilde{\mathcal{F}}_2$.
$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) = S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_2)$ with $A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_1) = A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_2)$ iff $\tilde{\mathcal{F}}_1 = \tilde{\mathcal{F}}_2$."

Example 1. Let $\tilde{\mathcal{F}}_1=\langle 0.7,0.6\rangle$ and $\tilde{\mathcal{F}}_2=\langle 0.8,0.5\rangle$ be two farmatean fuzzy sets (FFSs); then we have the following operation,
By using the score function $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}})=\frac{1}{2}\left(1+\alpha_{\tilde{\mathcal{F}}}{}^3-\beta_{\tilde{\mathcal{F}}}{}^3\right).(\min(\alpha_{\tilde{\mathcal{F}}},\beta_{\tilde{\mathcal{F}}}))$.

$$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1)=\frac{1}{2}(1+0.7^3-0.6^3).(\min(0.7,0.6))=0.337$$

$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_2) = \frac{1}{2}(1 + 0.8^3 - 0.5^3).(\min(0.8,0.5)) = 0.346$

Hence $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) < S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) \Rightarrow \tilde{\mathcal{F}}_1 < \tilde{\mathcal{F}}_2$.

Example 2. Let $\tilde{\mathcal{F}}_1 = \langle 0.9,0.8 \rangle$ and $\tilde{\mathcal{F}}_2 = \langle 0.6,0.5 \rangle$ be two farmatean fuzzy sets (FFSs); then we have the following operation, By using the score function $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}) = \frac{1}{2}\left(1 + \alpha_{\tilde{\mathcal{F}}}{}^3 - \beta_{\tilde{\mathcal{F}}}{}^3\right).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}))$.

$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) = \frac{1}{2}(1 + 0.9^3 - 0.8^3).(\min(0.9,0.8)) = 0.486$

$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_2) = \frac{1}{2}(1 + 0.6^3 - 0.5^3).(\min(0.6,0.5)) = 0.022$

Hence $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) > S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) \Rightarrow \tilde{\mathcal{F}}_1 > \tilde{\mathcal{F}}_2$.

IV. MATHEMATICAL MODEL

We use the notations mentioned below to create a mathematical model of the TP.

Notations

i: the origin, $\forall\, i = 1,2,\ldots,I$
j: the destination, $\forall\, j = 1,2,\ldots,J$
m: total number of sources,
n: total number of destinations,
c_{ij}: transportation cost per unit product from ith source to jth destination,
s_i: availability of goods at ith origin,
d_j: demand at jth destination,
x_{ij}: The amount of cargo that must be moved from ith source location to jth destination location
k: The number of conveyances,$\forall\, k = 1,2,\ldots,K$
t: The objective function, $\forall\, t = 1, 2, \ldots, T$
x_{ijk}: Number of goods transported by conveyance k from source i to destination j
s_i: The availability of goods at origin i (the total amount of items supplied by the source i, $\forall\, i = 1,2,\ldots,m$.)
d_j: Product demand at destination j (the total amount of items received by destination j, $\forall\, j = 1,2,\ldots,n$.)
e_k: Product shipment capacities of conveyance k
$C^{(t)}{}_{ijk}$: The unit transportation cost for the objective function at level t from the ith source to the jth destination via the kth mode of conveyance
$\bar{C}_{ijk}{}^{(t)}$: The unknown unit transportation cost for the objective function at level t from the ith source to the jth destination via the kth mode of conveyance
$\tilde{s}_i$: Unknown availability of items at i
$\tilde{d}_j$: Unknown requirement of items at j
$\tilde{e}_k$: Unknown items cargo capacity of the conveyance k

The formulation of the mathematical model of a traditional TP is defined as follows:

Model 4.1

Minimize $$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} \qquad (1)$$

Subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} \le s_i \; (i = 1,2,\ldots,m) \qquad (2)$$
$$\sum_{i=1}^{m} x_{ij} \ge d_j \; (j = 1,2.\ldots,n) \qquad (3)$$
$$x_{ij} \ge 0 \;\forall\; i \text{ and } j. \qquad (4)$$

According to constraint (1), the amount of goods that can be found at the source must be greater than or equal to the amount that needs to be delivered to the destinations from the source. Constraints (2) indicate that the number of items that must be delivered from the sources to the destination must satisfy the destination's minimum requirements. The constraint (3) demonstrates that the quantity of goods delivered cannot be less than zero.

Model 4.2

The mathematical model of traditional TP with FFP.

Minimize

$$z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}{}^{\tilde{\mathcal{F}}}\; x_{ij} \qquad (5)$$

Subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} \le s_i{}^{\tilde{\mathcal{F}}}, (i = 1,2,\ldots,m) \qquad (6)$$

$$\sum_{i=1}^{m} x_{ij} \geq d_j^{\tilde{\mathcal{F}}}, (j = 1,2.\dots, n) \tag{7}$$

Where

$$s_i^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{s}_i}, \beta_{\tilde{s}_i}) \text{ and } 0 \leq {\alpha_{\tilde{s}_i}}^3 + {\beta_{\tilde{s}_i}}^3 \leq 1,$$
$$b_j^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{b}_i}, \beta_{\tilde{b}_i}) \text{ and } 0 \leq {\alpha_{\tilde{b}_i}}^3 + {\beta_{\tilde{b}_i}}^3 \leq 1,$$
$$C_{ij}^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{c}_{ij}}, \beta_{\tilde{c}_{ij}}) \text{ and } 0 \leq {\alpha_{\tilde{c}_{ij}}}^3 + {\beta_{\tilde{c}_{ij}}}^3 \leq 1,$$
$$x_{ij} \geq 0 \ \forall \text{ i and j.} \tag{8}$$

Now convert the mathematical model 4.2 of the transportation problem with FFP into crisp form using NFFSF. The mathematical model 4.2 can be represented as follows.

Model 4.3
Minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} S(C_{ij}^{\tilde{\mathcal{F}}})\, x_{ij} \tag{9}$$

Subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} \leq S(s_i^{\tilde{\mathcal{F}}}), (i = 1,2,\dots, m) \tag{10}$$
$$\sum_{i=1}^{m} x_{ij} \geq S(d_j^{\tilde{\mathcal{F}}}), (j = 1,2.\dots, n) \tag{11}$$
$$x_{ij} \geq 0 \ \forall \text{ i and j.} \tag{12}$$

Assume a hierarchy-based decision-making system where there are t decision-makers (DM) and t objectives that must be satisfied. We once again developed the MOMLST model to use the above-mentioned notations.

Model 4.4
The mathematical model of the MOMLST problem for levels t is prescribed as follows,
Level 1

$$\mathrm{Min}_{\bar{x}_1} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} C^{(1)}{}_{ijk} x_{ijk} \tag{13}$$

Level 2
Where $\bar{x}_2$ solves

$$\mathrm{Min}_{\bar{x}_2} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} C^{(2)}{}_{ijk} x_{ijk} \tag{14}$$

: : :
: : :
: : :

Level t
Where $\bar{x}_t$ solves

$$\mathrm{Min}_{\bar{x}_t} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} C^{(t)}{}_{ijk} x_{ijk} \tag{15}$$

Subject to the constraints:

$$\sum_{j=1}^{J} \sum_{k=1}^{K} x_{ijk} \leq s_i, i = 1,2,\dots, I. \tag{16}$$
$$\sum_{i=1}^{I} \sum_{k=1}^{K} x_{ijk} \leq d_j, j = 1,2,\dots, J. \tag{17}$$
$$\sum_{i=1}^{I} \sum_{j=1}^{J} x_{ijk} \leq e_k, k = 1,2,\dots, K. \tag{18}$$
$$x_{ijk} \geq 0, i = 1,2,\dots, I \ \ j = 1,2,\dots, J \ \ k = 1,2,\dots, K. \tag{19}$$

Multiobjective multi-level uncertain solid transportation model:

Model 4.5
In (MOMLST) model, the cost of the objective function, availability, demand, and cargo capacity of the conveyance are uncertain in nature. The (MOMLUST) for levels 1, 2, ., t can be represented as follows,

Level 1

$$\mathrm{Min}_{\bar{x}_1} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} {\bar{C}_{ijk}}^{(1)} x_{ijk} \tag{20}$$

Level 2
Where $\bar{x}_2$ solves

$$\mathrm{Min}_{\bar{x}_2} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} {\bar{C}_{ijk}}^{(2)} x_{ijk} \tag{21}$$

: : :
: : :

: : :

Level t

Where $\bar{x}_t$ solves

$$\mathrm{Min}_{\bar{x}_t} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \bar{C}_{ijk}{}^{(t)} x_{ijk} \tag{22}$$

Subject to the constraints:

$$\sum_{j=1}^{J} \sum_{k=1}^{K} x_{ijk} \le \tilde{s}_i, i = 1,2,\ldots,I. \tag{23}$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} x_{ijk} \le \tilde{d}_j, j = 1,2,\ldots,J. \tag{24}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} x_{ijk} \le \tilde{e}_k, k = 1,2,\ldots,K. \tag{25}$$

$$x_{ijk} \ge 0, i = 1,2,\ldots,I \ j = 1,2,\ldots,J \ k = 1,2,\ldots,K. \tag{26}$$

Now, we convert model 4.5 by utilizing the expected value method in its predictable form because all the parameters involved in model 4.5 are uncertain in nature. Therefore, the deterministic model of MOMLUST is described as follows.

Model 4.6

The deterministic model of MOMLUST has been represented in this section.

Level 1

$$\mathrm{Min}_{\bar{x}_1} E(\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \bar{C}_{ijk}{}^{(1)} x_{ijk}) \tag{27}$$

Level 2

Where $\bar{x}_2$ solves

$$\mathrm{Min}_{\bar{x}_2} E(\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \bar{C}_{ijk}{}^{(2)} x_{ijk}) \tag{28}$$

: : :

: : :

: : :

Level t

Where $\bar{x}_t$ solves

$$\mathrm{Min}_{\bar{x}_t} E(\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \bar{C}_{ijk}{}^{(t)} x_{ijk}) \tag{29}$$

Subject to the constraints:

$$E(\sum_{j=1}^{J} \sum_{k=1}^{K} x_{ijk} - \tilde{s}_i) \le 0, i = 1,2,\ldots,I. \tag{30}$$

$$E(\sum_{i=1}^{I} \sum_{k=1}^{K} x_{ijk} - \tilde{d}_j \le 0, j = 1,2,\ldots,J. \tag{31}$$

$$E(\sum_{i=1}^{I} \sum_{j=1}^{J} x_{ijk} - \tilde{e}_k) \le 0, k = 1,2,\ldots,K. \tag{32}$$

$$x_{ijk} \ge 0, i = 1,2,\ldots,I \ j = 1,2,\ldots,J \ k = 1,2,\ldots,K. \tag{33}$$

Now, we apply the properties of the expected value approach in the above-mentioned model 4.6., the model MOMLUST has been represented given below.

Model 4.7

Level 1

$$\mathrm{Min}_{\bar{x}_1} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} E(\bar{C}_{ijk}{}^{(1)}) x_{ijk} \tag{34}$$

Level 2

Where $\bar{x}_2$ solves

$$\mathrm{Min}_{\bar{x}_2} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} E(\bar{C}_{ijk}{}^{(2)}) x_{ijk} \tag{35}$$

: : :

: : :

: : :

Level t

Where $\bar{x}_t$ solves

$$\mathrm{Min}_{\bar{x}_t} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} E(\bar{C}_{ijk}{}^{(t)}) x_{ijk} \tag{36}$$

Subject to the constraints:

$$\sum_{j=1}^{J} \sum_{k=1}^{K} x_{ijk} - E(\tilde{s}_i) \le 0, i = 1,2,\ldots,I. \tag{37}$$

$$E(\tilde{d}_j) - \sum_{i=1}^{I} \sum_{k=1}^{K} x_{ijk} \le 0, j = 1,2,\ldots,J. \tag{38}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} x_{ijk} - E(\tilde{e}_k) \le 0, k = 1,2,\ldots,K. \tag{39}$$

$$x_{ijk} \geq 0, i = 1,2,\dots,I \;\; j = 1,2,\dots,J \;\; k = 1,2,\dots,K. \tag{40}$$

The MOMLSTPWFF model:
The mathematical model MOMLSTPWFF, including the above-mentioned nomenclatures for levels $1,2,\dots,t$, is described below.

Model 4.8
Level 1

$$\mathrm{Min}_{\bar{x}_1} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} {C^{(1)}}_{ijk}^{\tilde{\mathcal{F}}} x_{ijk} \tag{41}$$

Level 2
Where $\bar{x}_2$ solves

$$\mathrm{Min}_{\bar{x}_2} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} {C^{(2)}}_{ijk}^{\tilde{\tilde{\mathcal{F}}}} x_{ijk} \tag{42}$$

$$\begin{matrix} \vdots & \vdots & \vdots \end{matrix}$$

Level t
Where $\bar{x}_t$ solves

$$\mathrm{Min}_{\bar{x}_t} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} {C^{(t)}}_{ijk}^{\tilde{\tilde{\mathcal{F}}}} x_{ijk} \tag{43}$$

Subject to the constraints

$$\sum_{j=1}^{J} \sum_{k=1}^{K} x_{ijk} \leq s_i^{\tilde{\mathcal{F}}}, i = 1,2,\dots,I. \tag{44}$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} x_{ijk} \leq d_j^{\tilde{\mathcal{F}}}, j = 1,2,\dots,J. \tag{45}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} x_{ijk} \leq e_k^{\tilde{\mathcal{F}}}, k = 1,2,\dots,K. \tag{46}$$

Where,

$$s_i^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{s}_i}, \beta_{\tilde{s}_i}) \text{ and } 0 \leq {\alpha_{\tilde{s}_i}}^3 + {\beta_{\tilde{s}_i}}^3 \leq 1,$$

$$d_j^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{d}_i}, \beta_{\tilde{d}_i}) \text{ and } 0 \leq {\alpha_{\tilde{b}_i}}^3 + {\beta_{\tilde{b}_i}}^3 \leq 1,$$

$$e_k^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{e}_k}, \beta_{\tilde{e}_k}) \text{ and } 0 \leq {\alpha_{\tilde{e}_k}}^3 + {\beta_{\tilde{e}_k}}^3 \leq 1,$$

$${C^{(t)}}_{ijk}^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{c}_{ij}}, \beta_{\tilde{c}_{ij}}) \text{ and } 0 \leq {\alpha_{\tilde{c}_{ij}}}^3 + {\beta_{\tilde{c}_{ij}}}^3 \leq 1$$

$$x_{ijk} \geq 0, i = 1,2,\dots,I, \;\; j = 1,2,\dots,J, \;\; k = 1,2,\dots,K. \tag{47}$$

The model mentioned above (41)– (47) is created assuming all relevant parameters are extending versions of fuzzy parameters. However, in the real world, there is always some degree of ambiguity, which complicates the model. It is currently considered that the problem's variables are all independent and fermatean fuzzy. The model mentioned above is known as MOMLSTPWFF. It cannot be directly optimized because the FFP exists in subject to the constraints and objective functions. However, we will follow the new score function in FFSs and convert model 4.8 into crisp form. Model 4.8 can be restated by considering the anticipated values of subject to the constraints and objective functions for $1,2,\dots,t$, levels.

Model 4.9
Level 1

$$\mathrm{Min}_{\bar{x}_1} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} S({C^{(1)}}_{ijk}^{\tilde{\mathcal{F}}})\, x_{ijk} \tag{48}$$

Level 2
Where $\bar{x}_2$ solves

$$\mathrm{Min}_{\bar{x}_2} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} S({C^{(2)}}_{ijk}^{\tilde{\tilde{\mathcal{F}}}})\, x_{ijk} \tag{49}$$

$$\begin{matrix} \vdots & \vdots & \vdots & \vdots \end{matrix}$$

Level t
Where $\bar{x}_t$ solves

$$\mathrm{Min}_{\bar{x}_t} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} S({C^{(t)}}_{ijk}^{\tilde{\tilde{\mathcal{F}}}})\, x_{ijk} \tag{50}$$

Subject to the constraints:

$$\sum_{j=1}^{J} \sum_{k=1}^{K} x_{ijk} \leq S(s_i^{\tilde{\mathcal{F}}}), i = 1,2,\dots,I. \tag{51}$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K} x_{ijk} \leq S(d_j^{\tilde{\mathcal{F}}}), j = 1,2,\ldots,J. \quad (52)$$
$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk} \leq S(e_k^{\tilde{\mathcal{F}}}), k = 1,2,\ldots,K. \quad (53)$$
$$x_{ijk} \geq 0, i = 1,2,\ldots,I,\ j = 1,2,\ldots,J,\ k = 1,2,\ldots,K.) \quad (54)$$

V. SOLUTION METHODOLOGY

We developed a methodology for dealing with MOMLSTPWFF problems in an FFE. The suggested method includes the following steps:

Step 1: First, we create a multiobjective balancing solid transportation issue in which the total production and overall demand are equal.
Step 2: Then, convert the MOMLSTPWFF problems into crisp form using NFFSF with FFE.
Step 3: At this point, deal with this problem for all objectives individually. We obtain possible primary responses for every objective function.
Step 4: Determine the entire cost and duration of the transportation and deterioration cost during transportation.
Step 5: Using the Lingo 20.0 software package, solve the transportation problem using Fermatean fuzzy programming methodology.

VI. ILLUSTRATION OF MOMLSTPWFF

The numerical example is provided here to justify the convenience of MOMLSTPWFF. Finding a transportation planning strategy that is best for the MOMLSTPWFF. We want to optimize the best solution of three objective functions, viz., $Z_1{}^*$= Minimize total transportation costs, $Z_2{}^*$= Minimize total transportation time, $Z_3{}^*$=Minimize the deterioration cost during transportation. Each parameter used in the MOMLSTPWFF problem is an FFN in FFE. To accomplish this, the fermatean fuzzy programming approach has been implemented to examine the uncertainty. In MOMLSTPWFF, the FFN's total transportation cost, total transportation time, and deterioration cost during transportation for each unit of items from i source to j destinations for $i = 1,2,3$ and $j = 1,2,3,4$ are obtained solution using the fermatean fuzzy programming with NFFSF. However, both the supply and demand and the cost of objectives are measured using NFFSF.

The Fermatean fuzzy data are provided below and converted to deterministic form using the NFFSF $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{1D}) = \frac{1}{2}(1 + \alpha_{\tilde{\mathcal{F}}} - \beta_{\tilde{\mathcal{F}}}).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}))^2$.

Table 3: Fermatean Fuzzy Total Transportation Cost

Source	b_1	b_2	b_3	b_4
a_1	(0.4, 0.9)	(0.3, 0.8)	(0.1, 0.6)	(0.2, 0.99)
a_2	(0.7, 0.8)	(0.1, 0.9)	(0.2, 0.6)	(0.2, 0.1)
a_3	(0.5, 0.8)	(0.7, 0.99)	(0.1, 0.8)	(0.7, 0.9)

Table 4: Fermatean Fuzzy Total Transportation Time

Source	b_1	b_2	b_3	b_4
a_1	(0.5, 0.9)	(0.7, 0.5)	(0.2, 0.9)	(0.6, 0.9)
a_2	(0.6, 0.4)	(0.1, 0.99)	(0.6, 0.8)	(0.4, 0.7)
a_3	(0.3, 0.8)	(0.8, 0.6)	(0.5, 0.1)	(0.3, 0.9)

Table 5: Fermatean Fuzzy Deterioration Cost During Transportation

Source	b_1	b_2	b_3	b_4
a_1	(0.1, 0.7)	(0.3, 0.5)	(0.2, 0.7)	(0.8, 0.7)
a_2	(0.2, 0.8)	(0.1, 0.9)	(0.8, 0.1)	(0.4, 0.7)
a_3	(0.8, 0.1)	(0.7, 0.8)	(0.4, 0.9)	(0.5, 0.9)

Table 6: Fermatean Fuzzy Supply

i	a_1	a_2	a_3
$(\alpha_{\tilde{\mathcal{F}}_i}, \beta_{\tilde{\mathcal{F}}_i})$	(0.6, 0.4)	(0.4, 0.9)	(0.6, 0.5)

Table 7: Fermatean Fuzzy Demand

j	b_1	b_2	b_3	b_4
$(\alpha_{\tilde{\mathcal{F}}_j}, \beta_{\tilde{\mathcal{F}}_j})$	(0.3, 0.2)	(0.4, 0.8)	(0.4, 0.5)	(0.7, 0.5)

Now, we use the NFFSF $S_{\tilde{\mathcal{F}}}{}^{*}\left(\tilde{\mathcal{F}}_{1D}\right) = \frac{1}{2}(1 + \alpha_{\tilde{\mathcal{F}}} - \beta_{\tilde{\mathcal{F}}}).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}))^2$ in fermatean fuzzy programming, the fermatean fuzzy total transportation cost, total transportation time, deterioration cost during transportation availability and demand to convert all data as mentioned above into the crisp form. The crisp data are presented as follows;

Table 8: Total Transportation Cost With Crisp Form

Source	b_1	b_2	b_3	b_4
a_1	(0.04)	(0.0225)	(0.0025)	(0.0042)
a_2	(0.2205)	(0.001)	(0.012)	(0.0055)
a_3	(0.0875)	(0.1739)	(0.0015)	(0.196)

Table 9: Total Transportation Time With Crisp Form

Source	b_1	b_2	b_3	b_4
a_1	(0.075)	(0.15)	(0.006)	(0.126)
a_2	(0.096)	(0.00055)	(0.144)	(0.056)
a_3	(0.0225)	(0.216)	(0.007)	(0.018)

Table 10: Deterioration Cost During Transportation With Crisp Form

Source	b_1	b_2	b_3	b_4
a_1	(0.002)	(0.036)	(0.01)	(0.2695)
a_2	(0.008)	(0.001)	(0.0085)	(0.056)
a_3	(0.0085)	(0.2205)	(0.04)	(0.075)

Table 11: Supply in Crisp form

i	a_1	a_2	a_3
$(\alpha_{\tilde{\mathcal{F}}_i}, \beta_{\tilde{\mathcal{F}}_i})$	(0.096)	(0.2025)	(0.1375)

Table 12: Demand in Crisp form

j	b_1	b_2	b_3	b_4
$(\alpha_{\tilde{\mathcal{F}}_j}, \beta_{\tilde{\mathcal{F}}_j})$	(0.022)	(0.072)	(0.192)	(0.15)

Since total supply is equal to total demand (0.436), then this problem is called the multiobjective balance transportation problem. Using LINGO-20.0 software, the optimum solution has been determined to be as follows. The minimization of transportation costs ($Z_1{}^{*}$) is = 0.00167065, and minimization of total transportation time ($Z_2{}^{*}$) is 0.0094416, and the minimization of deterioration during transportation ($Z_3{}^{*}$) is 0.0126295.

Table 13: Solution of Minimization of Total Transportation Cost

Efficient Transportation Planning	Optimal Objective Value's ($Z_1{}^{*}$)
$x_{13} = 0.0545, 4 = 0.0415, x_{21} = 0.022, x_{22} = 0.072,$ $x_{24} = 0.1085, x_{33} = 0.1375$	0.00167065

Table 14: Solution of Minimization of Total Transportation Time

Efficient Transportation Planning	Optimal Objective Value's ($Z_2{}^{*}$)
$x_{13} = 0.096, x_{22} = 0.072, x_{24} = 0.1305, x_{31} = 0.022,$ $x_{33} = 0.096, x_{34} = 0.0195$	0.0094416

Table 15: Solution of Minimization the Deterioration During Transportation

Efficient Transportation Planning	Optimal Objective Value's ($Z_3{}^{*}$)
$x_{13} = 0.096, x_{22} = 0.072, x_{23} = 0.096, x_{24} = 0.0345,$ $x_{31} = 0.022, x_{34} = 0.1155$	0.0126295

VII. CONCLUSION

This paper investigates a novel category of multiobjective, multi-level mathematical models known as the solid transportation framework with multiple goals under uncertain parameters and the multiobjective multi-level solid transportation model with FFP. To begin with, we convert these models into deterministic models using the expected value approach and new score fermatean fuzzy programming approach with FFE. Additionally, the MOMLSTPWFF model is treated as a single-purpose role model to minimize three distinct objectives: total transportation costs, total transportation time, and the cost associated with transportation deterioration. The best way to address the transportation issue is to determine it using Lingo

20 software. A numerical example is also presented in this research to validate the effectiveness of this mathematical programming approach.

Competing Interests:

We would like to declare that we have no conflicts of interest that could potentially influence the publication of this research work.

Table 1: List of Abbreviations

Abbreviations	Full name
STP	Solid Transportation Problem
TP	Transportation Problem
NWCM	Northwest Corner Method
LCM	Least Cost Method
VAM	Vogel's Approximation Method
MOSTP	Multiobjective STP
FFSs	Fermatean Fuzzy Sets
MOTP	Multiobjective Transportation Problem
FFNs	Fermatean Fuzzy Numbers
MCDM	Multi-criteria Decision Making
MOGSTLP	Multiobjective Green Solid Transportation-Location Problem
MSMOFSTP	Multi-Stage Multiobjective Fixed-charge STP
MOGFDFTP	Multiobjective Green Four Dimensional Fixed- charge Transportation Problem
FMOTP	Fuzzy Multiobjective Transportation Problem
DEA	Data Envelopment Analysis
IVIF	Interval-Valued Intuitionistic Fuzzy Sets
FFMOTP	Fermatean Fuzzy Multiobjective Transportation Problem
MOMLSTPWFF	Multiobjective Multi-Level STP With Fermatean Fuzzy
MOMLST	Multiobjective Multi-Level STP
MOMLUST	Multiobjective Multi-Level Uncertain STP

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